Measuring photon anti-bunching from continuous variable sideband squeezing

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We present a technique for measuring the second-order coherence function $g^{(2)}(\tau)$ of light using a Hanbury-Brown Twiss intensity interferometer modified for homodyne detection. The experiment was performed entirely in the continuous variable regime at the sideband frequency of a bright carrier field. We used the setup to characterize $g^{(2)}(\tau)$ for thermal and coherent states, and investigated its immunity to optical loss. We measured $g^{(2)}(\tau)$ of a displaced squeezed state, and found a best anti-bunching statistic of $g^{(2)}(0) = 0.11 \pm 0.18$.

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Fifty years ago, Hanbury-Brown and Twiss (HBT) first demonstrated an optical intensity interferometer [1]. Since then, HBT interferometry has been applied to diverse areas such as condensed matter physics, atomic physics, and quantum optics [2, 3, 4]; and has become a powerful measurement technique in astronomy, and high-energy particle physics [5, 6]. From a historical perspective, HBT reported correlations in the intensity measured at two locations, from light emitted by a thermal source. The effect was interpreted as being either a manifestation of classical wave theory, or due to *bunching* in the arrival time of photons. Such correlations were generalized to *n*th-order by Glauber in a comprehensive quantum theory of optical coherence [7], with the second-order coherence $g^{(2)}$ corresponding to the measurement made with a HBT interferometer. Curiously, the theory predicted that certain states of light would exhibit a photon *anti-bunching* effect, which is the tendency for photons to arrive apart from one another. This is a non-classical phenomenon which violates the Schwartz inequality [4]. Photon anti-bunching has been observed in resonance fluorescence, conditioned measurements of parametrically down-converted light, pulsed parametric amplification, quantum dots, and trapped single atoms/molecules [8, 9, 10, 11, 12]. Recent experiments have probed the spatial and temporal second-order coherence functions of atomic species in BEC, and atom lasers [13, 14]. All of these experiments have relied upon the ability to detect individual particles in a time-resolved measurement.

In this Letter we apply a technique for measuring the second-order coherence of optical fields, that complements previous studies and provides a link between discrete-variable (DV) and continuous-variable (CV) quantum optics. Our scheme is based on the HBT interferometer, but uses homodyne detection in each arm, to make CV measurements of the quadrature amplitudes over a range of sideband frequencies. The second-order coherence $g^{(2)}$ is then constructed from the set of four permutations of the time-averaged correlations between the amplitude/phase quadratures. At no point is it necessary to make time-resolved detections of single-photons [15]. Homodyne detection offers the advantage of high bandwidth, and excellent immunity to extraneous optical modes. We used the scheme to measure the temporal second-order coherence function $g^{(2)}(\tau)$ of a displaced squeezed state.

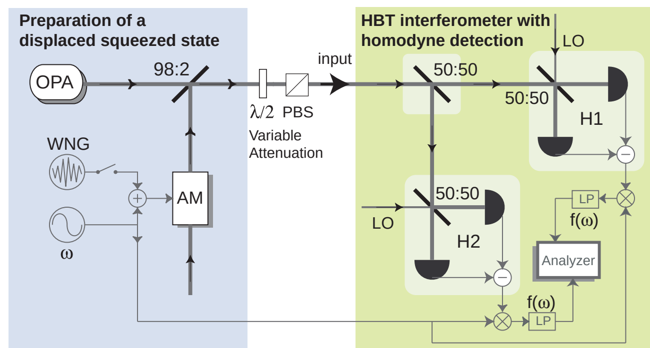

FIG. 1: (Color online) Schematic of the experimental setup. OPA optical parametric amplifier, $\lambda/2$ half-wave plate, PBS polarizing beam-splitter, x:y beamsplitter with transmission x, H1/H2 homodyne detectors, AM amplitude modulator, ω function generator, WNG white noise generator, $\otimes$ mixer, LP low pass filter.

In contrast to most CV experiments involving squeezed light, is the realization that weaker squeezed states can exhibit a greater anti-bunching effect. Some properties of displaced squeezed states in the context of second-order coherence have been investigated before [10, 16]. Such states can exhibit behavior ranging from photon anti-bunching to super-bunching, provided that the state is sufficiently pure, and the squeezing weak. Using our modified HBT interferometer, and exploiting the high stability and low optical loss of our experimental setup, we were able to prepare and measure displaced squeezed states that clearly demonstrated photon anti-bunching. In addition, we investigated the immunity of second-order coherence measurements to optical attenuation.

Theory: The second-order temporal coherence function allows the investigation of photon grouping effects in terms of normally ordered intensity-intensity correlations. The normalized temporal coherence function of a stationary radiation field consisting of a single mode $\hat{a}$ is defined as the probability of a joint detection of two photons at delayed times t and $t+\tau$

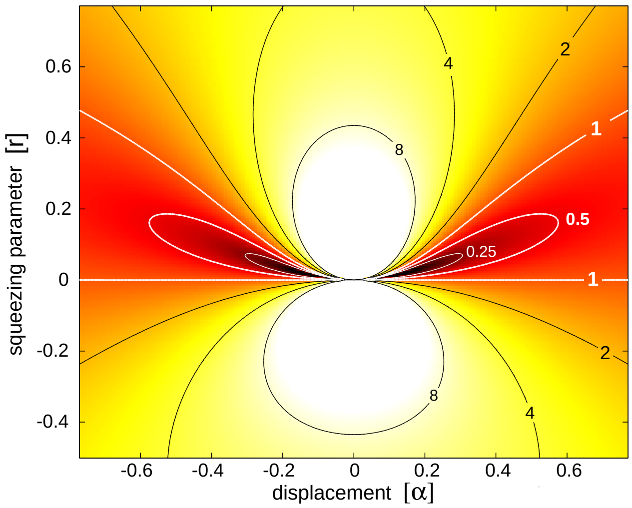

FIG. 2: (Color online) The $g^{(2)}(0)$ parameter space for a displaced squeezed state with squeezing r and displacement α. Regions exhibiting photon anti-bunching are marked progressively darker, with contours giving the precise values.

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t+\tau)\hat{a}^\dagger(t)\hat{a}(t+\tau)\hat{a}(t)\rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle^2}. \qquad (1)$$

For classical fields the operators are replaced by complex numbers, and reordered such that at no delay we can write $g^{(2)}(0) = \langle I^2\rangle/\langle I\rangle^2$ with I the field intensity. This quantity is bounded below by one, so classical fields cannot display anti-bunching. The non-commutation of $\hat{a}$ and $\hat{a}^\dagger$ prevents the quantum expression from being reordered in this way, and allows for values of $g^{(2)}$ less than one. It is possible to rewrite Eq. (1) as a product of intensities after a beamsplitter, such as measured on the HBT interferometer

$$g^{(2)}(\tau) = \frac{\langle \hat{b}^\dagger(t+\tau)\hat{b}(t+\tau)\hat{c}^\dagger(t)\hat{c}(t)\rangle}{\langle \hat{b}^\dagger(t)\hat{b}(t)\rangle\langle \hat{c}^\dagger(t)\hat{c}(t)\rangle}. \qquad (2)$$

If $\hat{b} = (\hat{a}+\hat{v})/\sqrt{2}$ and $\hat{c} = (\hat{a}-\hat{v})/\sqrt{2}$ with $\hat{v}$ a vacuum mode, then Eq. (1) and Eq. (2) are formally equivalent. The temporal coherence function witnesses the anti-bunching effect when $g^{(2)}(0) < g^{(2)}(\tau\neq 0)$. The opposite inequality holds true for photon bunching. This method of measuring $g^{(2)}(\tau)$ is possible only when one has access to the photon number correlations directly.

For experiments in the CV regime, a connection must be found for measurements made via homodyne detection. A pair of homodyne detectors will measure correlations between the field amplitude $\hat{X}_a^+ = \hat{a}+\hat{a}^\dagger$, or phase $\hat{X}_a^- = -i(\hat{a}-\hat{a}^\dagger)$, quadratures. Therefore we re-express the coherence function Eq. (2) in terms of quadrature operators

$$g^{(2)}(\tau) = \frac{\sum_{i,j}\langle \hat{X}_b^i(t+\tau)^2\hat{X}_c^j(t)^2\rangle - 2\sum_{i,k}\langle \hat{X}_k^i(t)^2\rangle + 4}{(\sum_i\langle \hat{X}_b^i(t)^2\rangle - 2)(\sum_i\langle \hat{X}_c^i(t)^2\rangle - 2)} \qquad (3)$$

with $i,j = +,-$ and $k = b,c$. What makes this measurement technique possible is that in the equation there are no cross-quadrature correlation terms for a single mode. Hence each correlation or variance term can be measured independently by recording the output of the homodyne detectors (H1/H2 in Fig. 1), and a value for $g^{(2)}(\tau)$ constructed accordingly.

We wish to consider the coherence function Eq. (2) for a weakly squeezed and weakly displaced vacuum state. We work in the Heisenberg picture. The operator $\hat{a}$ is first transformed according to

$$\hat{D}^\dagger(\alpha)\hat{S}^\dagger(r)\hat{a}\hat{S}(r)\hat{D}(\alpha) = \alpha + \hat{a}\cosh r - \hat{a}^\dagger \sinh r \qquad (4)$$

where $\hat{D}$ and $\hat{S}$ are the unitary displacement and squeezing operators respectively [4], and we choose $\alpha \in \Re$ and $r \in \Re$. Then by using Eq. (4) in Eq. (2) with $\tau = 0$ we find

$$g^2(0) = 1 + \frac{\sinh^2(r)\left(2\alpha^2 + \cosh(2r) - 2\alpha^2\coth(r)\right)}{\left(\alpha^2+\sinh^2(r)\right)^2}. \qquad (5)$$

This function is plotted in Fig. 2 where bunching or anti-bunching statistics can be found with the correct choice of r and α. The stronger cases occur for states approaching the vacuum state $\alpha = r = 0$ for which $g^{(2)}(0)$ is undefined. Indeed it's possible to approach the vacuum state while holding any value of $g^{(2)}(0)$ constant. The expression for $g^{(2)}(0)$ can be extended to arbitrary τ by repeating the derivation but including a delay of τ on mode $\hat{b}$ we obtain

$$\begin{aligned} g^{(2)}(\tau) = &\frac{1}{(\sinh^2(r)+\alpha^2)^2}\Big\{\Big(\alpha^2 - \frac{1}{2}[\hat{a}(\tau),\hat{a}^\dagger(0)]\sinh(2r)\Big)^2 \\ &+ \big(2+[\hat{a}(0),\hat{a}^\dagger(\tau)]+[\hat{a}(\tau),\hat{a}^\dagger(0)]\big)\alpha^2\sinh^2(r) \\ &+ \big(1+[\hat{a}(0),\hat{a}^\dagger(\tau)]^2\big)\sinh^4(r)\Big\}. \end{aligned} \qquad (6)$$

Here, the commutation relations depend on the shape of the filter used in the measurement process. The filter selects a particular frequency mode given by

$$\hat{a}(\tau) = N^{-\frac{1}{2}}\int_{-\infty}^{\infty} \hat{a}_\omega f(\omega) e^{i\tau\omega} d\omega \qquad (7)$$

where $f(\omega)$ is the filter function and $N = \int_{-\infty}^{\infty} f(\omega)^2 d\omega$ is a normalization factor. Using $[\hat{a}_\omega, \hat{a}_{\omega'}^\dagger] = \delta(\omega-\omega')$ and choosing the filter $f(|\omega| \leqslant \Omega) = 1$ and zero elsewhere gives the commutation relations $[\hat{a}(0),\hat{a}^\dagger(\tau)] = [\hat{a}(\tau),\hat{a}^\dagger(0)] = \mathrm{sinc}(\Omega\tau)$. Substituting these into Eq. (6) gives

$$\begin{aligned} g^{(2)}(\tau) = &\frac{1}{(\sinh^2(r)+\alpha^2)^2}\Big\{\Big(\alpha^2 - \frac{1}{2}\mathrm{sinc}(\Omega\tau)\sinh(2r)\Big)^2 \\ &+ 2\left(1+\mathrm{sinc}(\Omega\tau)\right)\alpha^2\sinh^2(r) \\ &+ \left(1+\mathrm{sinc}^2(\Omega\tau)\right)\sinh^4(r)\Big\}. \end{aligned} \qquad (8)$$

This expression is specific to *pure* states, but can be generalized for any Gaussian state having variances $V_{\mathrm{in}}^+, V_{\mathrm{in}}^-$ and mean α_{in} as measured at the input to the HBT interferometer

(see Fig. 1). The general form is

$$g^{(2)}(\tau) \;=\; 1+16\mathrm{sinc}(\Omega\tau)\frac{(V_{\rm in}^{+}-1)\alpha_{\rm in}^{2}}{(2-V_{\rm in}^{+}-V_{\rm in}^{-}-4\alpha_{\rm in}^{2})^{2}} + 2\mathrm{sinc}^{2}(\Omega\tau)\frac{2+(V_{\rm in}^{-}-2)V_{\rm in}^{-}+(V_{\rm in}^{+}-2)V_{\rm in}^{+}}{(2-V_{\rm in}^{+}-V_{\rm in}^{-}-4\alpha_{\rm in}^{2})^{2}}. \quad (9)$$

This equation was used for making theoretical predictions that were based on measurements $\{V_{\rm in}^{+}; V_{\rm in}^{-}; \alpha_{\rm in}\}$. A comparison with the actual measured $g^{(2)}(\tau)$ could then be made.

Experiment: The schematic shown in Fig. 1 includes: the source of squeezed light and displacement operation; the modified HBT interferometer with homodyne detection and data acquisition. A Nd:YAG laser emitted 1.5 W of coherent light at 1064 nm with a line-width of 1 kHz. A major fraction of this light was frequency-doubled and used to pump an optical parametric amplifier (OPA) composed of a $\mathrm{MgO:LiNbO_3}$ crystal. The OPA could be controlled so as to de-amplify the transmitted seed beam by gain factors from 0.9 to 0.5. This produced a bright beam ($\approx 5\ \mu$W) having broadband squeezing of the amplitude quadrature covering 3 MHz up to the cavity line-width of 15 MHz. Corresponding levels of squeezing were measured (at 6MHz) from $V_{\rm in}^{+}=0.89$ to $V_{\rm in}^{+}=0.55$, with values of purity $(V_{\rm in}^{+}V_{\rm in}^{-})$ ranging from 1.005 to 1.18, respectively. The displacement operation was performed by injecting a bright amplitude-modulated field at the 98:2 beam-splitter. The relative phase at the beam-splitter was controlled, so that the quadrature of the squeezing was preserved.

With the desired displaced squeezed state prepared, it was then subjected to the second-order coherence measurement process as performed via the modified HBT interferometer. The prepared state was mixed with a vacuum state on a 50:50 beam-splitter. Each of the two resulting output beams were sent on toward a balanced homodyne detector, at whose core was a pair of matched photodiodes (ETX500) with quantum efficiency estimated at $93\pm3\%$. Care was taken to keep the optical path lengths equal. The local-oscillators (LO) were mode-matched into the test beam with 96% visibility, and the homodyne condition fulfilled by a signal/LO power ratio of $1/1000$. The total detection efficiency was 86%. The LO phase was controlled, so as to measure the required quadrature of either amplitude or phase. The electronic signals of the photodetectors were amplified, mixed-down at 6 MHz, and low-pass filtered, before being over-sampled by a 12-bit digital-to-analogue converter at the rate of 240 kS/s. The time series data was then low-passed with a digital top-hat filter and down-sampled to 120 kS/s, thereby ensuring a flat power-spectrum with bandwidth 60 kHz. (This filter shape determines the final $g^{(2)}(\tau)$ function). The variances and correlation coefficients of the four permutations of the quadrature measurements were calculated using approximately one million data points acquired over 10 successive runs. These values were entered into Eq. (3) to get $g^{(2)}(\tau)$. The uncertainty in each measurement (68% confidence interval) was calculated using standard methods of error-analysis.

In this experiment we had the ability to freely vary the amount of displacement, and also add broad-band noise to the amplitude quadrature which simulated a thermal state. The

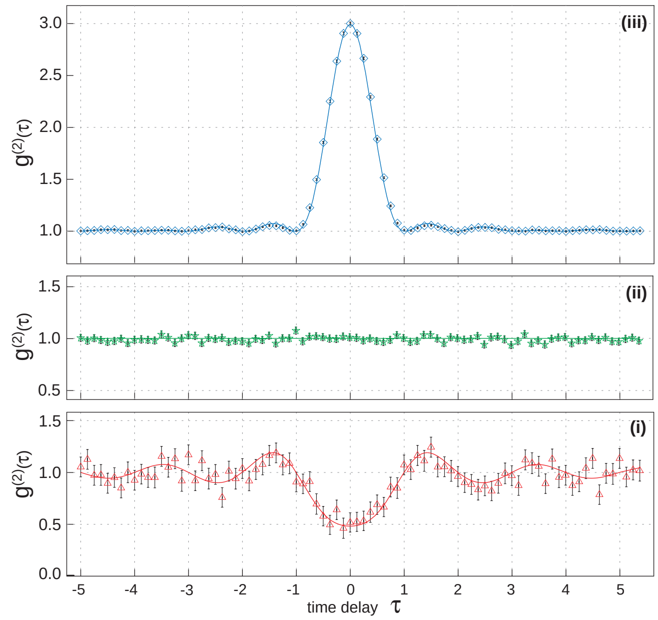

FIG. 3: (Color online) Experimental measurement of $g^{(2)}(\tau)$ with time delay τ in units of bandwidth $(\tau\Omega)$. (i) displaced squeezed state, (ii) coherent state, (iii) biased thermal state, curves are theoretical predictions.

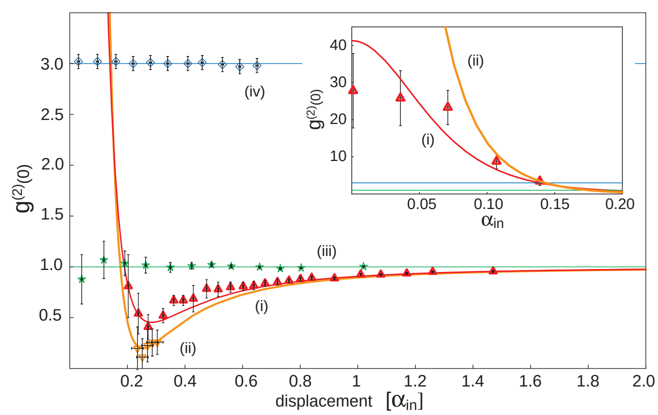

FIG. 4: (Color online) Main figure and inset: Experimental measurement of $g^{(2)}(0)$ as a function of displacement $\alpha_{\rm in}$. (i) displaced squeezed state, (ii) weak displaced squeezed state, (iii) coherent state, (iv) biased thermal state, curves are theoretical predictions.

squeezing strength and purity were linked together by the optical loss and seed-coupling mechanisms of the OPA, but some selection was possible by independent variation of the pump and seed powers of the OPA. Additional optical loss could be introduced by way of a variable optical-attenuator placed before the HBT interferometer. Finally, the electronic signals of each homodyne detector could be temporally shifted with respect to one another, both before and after digital sampling.

Results: Photon anti-bunching statistics from a displaced squeezed state were confirmed by the experimental results. Fig. 3(i) shows how the measured value of $g^{(2)}(0)$ varied as a function of time delay τ. In the experiment, a squeezed state

was prepared and measured to have $\{V_{\rm in}^{+}=0.902(1);V_{\rm in}^{-}=1.137(1)\}$. The state was then displaced by $\alpha_{\rm in}=0.257(1)$ which was the amount predicted to minimize $g^{(2)}$ for those particular variances. This prepared state yielded $g^{(2)}(0)=0.44(22)$ for zero time delay. Then as τ was increased, $g^{(2)}(\tau)$ approached unity. Thereby fulfilling the requirement for photon anti-bunching $g^{(2)}(0)<g^{(2)}(\tau)$.

Displaced squeezed states present their most interesting behavior when $g^{(2)}(0)$ is plotted along a range of displacements. Here, the state can easily be manipulated into exhibiting either anti-bunching or super-bunching statistics, depending only on the amount of displacement applied. The experimental data is plotted in Fig. 4(i), together with the theoretical curve as determined solely by $\{V_{\rm in}^{+};V_{\rm in}^{-}\}$. Super-bunching statistics were measured (see inset of Fig. 4) from the prepared state $\{V_{\rm in}^{+}=0.901(3);V_{\rm in}^{-}=1.136(1)\}$ with zero displacement $\alpha_{\rm in}=0.001(2)$ which produced $g^{(2)}(0)=28(10)$. On increasing $\alpha_{\rm in}$ still further, the state first found a minimum value corresponding to anti-bunching $g^{(2)}(0)=0.41(12)$ and then monotonically approached one.

Aiming to observe even stronger anti-bunching, we prepared a squeezed state approaching higher purity $\{V_{\rm in}^{+}V_{\rm in}^{-}=0.890(2)\times 1.129(2)=1.005(3)\}$. This near pure state was produced when the seed power to the OPA was reduced, which further de-coupled extraneous noise sources. The $g^{(2)}(0)$ of this state was measured for a range of displacements; Fig. 4(ii). The minimum value was found at $\alpha_{\rm in}=0.252(2)$ which yielded $g^{(2)}(0)=0.11(18)$. For comparison, a two-photon Fock state would be limited to $g^{(2)}(0)$=0.5.

To fully characterize our HBT interferometer, we used two test states that could readily be produced, and whose properties were well known; the coherent state and the biased thermal-state (both with variable displacement). Theory predicts that a coherent state will produce $g^{(2)}(\tau)=1$ independent of $\alpha_{\rm in}$. This was confirmed for $\tau=0$ in the measurement Fig. 4(iii), and also for variable τ in Fig. 3(ii). Both sets of measurements kept within $g^{(2)}(\tau)=1.00(6)$.

A biased thermal-state was made by taking a coherent state, and applying to only the amplitude quadrature, a displacement driven by a broad-band white-noise source. The state was called biased because $\{V_{\rm in}^{+}>1;V_{\rm in}^{-}=1\}$. The prediction for a strongly biased thermal-state is $g^{(2)}(0)\simeq 3$ for $V_{\rm in}^{+}\gg 1$ and small displacments $(\alpha_{\rm in}\sim 1)$. We prepared a state with $\{V_{\rm in}^{+}=12.80(9);V_{\rm in}^{-}=1.039(1)\}$ and varied $\alpha_{\rm in}$ from zero to $0.65(1)$ to obtain Fig. 4(iv). The result was $g^{(2)}(0)=2.98(1)$ which adhered to the theoretical prediction. This state was also studied under variable time delay; Fig. 3(iii). At τ=0, the function was maximum $g^{(2)}(0)=2.98(1)$, and then fell towards one with increasing τ, according to the sinc function. The prepared state was $\{V_{\rm in}^{+}=14.60(2);V_{\rm in}^{-}=1.025(8);\alpha_{\rm in}=0.258(1)\}$. The results from both the coherent state and biased-thermal state measurements agreed with the theoretical predictions, and validated our HBT interferometer.

A curious property of the $g^{(2)}$ measurement is that its value is unchanged by prior mixing of the state with a vacuum state, hence $g^{(2)}$ measurements are invariant to optical loss. We tested this property by preparing a displaced-squeezed state $\{V_{\rm in}^{+}=0.894(2);V_{\rm in}^{-}=1.139(2);\alpha_{\rm in}=0.255(2)\}$ and passing it through variable optical attenuation. With zero attenuation, and total detection efficiency $\eta_{\rm det}$=86% we measured $g^{(2)}(0)=0.67(16)$. Increasing the attenuation to $\eta_{\rm det}$=43% yielded $g^{(2)}(0)$=0.43(36). This showed that within a confidence interval, $g^{(2)}$ was invariant to optical loss. The invariance comes at the cost of increasing the uncertainty in the measurement. In principle, the total measurement time can be increased to accommodate an arbitrary level of attenuation.

In summary, we have used a continuous-variable measurement scheme to experimentally probe the second-order coherence function of quantum states of light. The scheme was based on the Hanbury-Brown Twiss intensity interferometer, but used homodyne detection of the quadrature amplitudes, in contrast to discrete-variable single photon counting. By preparing displaced squeezed states with the appropriate parameters, we were able to observe photon anti-bunching as witnessed by $g^{(2)}(0)$=0.11(18), which is a non-classical effect, and a direct manifestation of the quantum nature of light.

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